Problem I–1

Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that

$$f\big(xf(y) + 2y\big) = f(xy) + xf(y) + f\big(f(y)\big)$$

holds for all real numbers x and y.

Problem I–2

Let $n \geq 3$ be an integer. We say that a vertex A_i $(1 \leq i \leq n)$ of a convex polygon $A_1A_2\dots A_n$ is *Bohemian* if its reflection with respect to the midpoint of the segment $A_{i-1}A_{i+1}$ (with $A_0 = A_n$ and $A_{n+1} = A_1$) lies inside or on the boundary of the polygon $A_1A_2\dots A_n$. Determine the smallest possible number of Bohemian vertices a convex n-gon can have (depending on n).

(A convex polygon $A_1A_2\dots A_n$ has n vertices with all inner angles smaller than $180°$.)

Problem I–3

Let ABC be an acute-angled triangle with $AC > BC$ and circumcircle ω. Suppose that P is a point on ω such that $AP = AC$ and that P is an interior point of the shorter arc BC of ω. Let Q be the point of intersection of the lines AP and BC. Furthermore, suppose that R is a point on ω such that $QA = QR$ and that R is an interior point of the shorter arc AC of ω. Finally, let S be the point of intersection of the line BC with the perpendicular bisector of the side AB. Prove that the points P, Q, R, and S are concyclic.

Problem I–4

Determine the smallest positive integer n for which the following statement holds true: From any n consecutive integers one can select a non-empty set of consecutive integers such that their sum is divisible by 2019.

Time: 5 hours
Time for questions: 60 min
Each problem is worth 8 points.
The order of the problems does not depend on their difficulty.